Early Modern Sanskrit Logic and Self-Reference

Circularity in Navya-Nyāya Logic

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Aims of the Presentation

- Demonstrate competence in Navya-Nyāya logic.
- Demonstrate an understanding of the formal techniques used to symbolize the logic.
- Demonstrate an understanding of the philosophical history of the logic.

Recap of Paradox

How is a paradox different from a contradiction?

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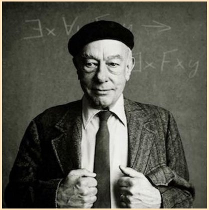
A paradox is the distinct assertion of a statement and its contrary. A contradiction is the assertion of a known falsity such as a conjunction of contraries.

Recap of Paradox

"A veridical paradox packs a surprise, but the surprise quickly dissipates itself as we ponder the proof. A falsidical paradox packs a surprise, but is seen as a false alarm when we solve the underlying fallacy. An antinomy, however, packs a surprise that can be accommodated by nothing less than a repudiation of our conceptual heritage." *(The Ways of Paradox, WVO Quine)*



WVO Quine

Recap of Inclosure

Inclosure Schema

Let ϕ and ψ be two predicates, δ be a possibly partial function. The following two conditions are mutually inconsistent:

1. Existence - $\Omega = \{y; \phi(y)\}$ exists and $\psi(\Omega)$.
2. (a) Transcendence - if $S \subseteq \Omega$ and $\psi(S)$, then $\delta(S) \notin S$.
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Russell's Paradox
Consider $\phi(x)$ as the predicate "$x \notin x$". $\psi(y)$ is the universal predicate true of anything. δ is the identity function. Then,

- $\Omega = \{x | x \notin x\}$ exists and $\psi(\Omega)$ holds.
- Consider the case $\Omega \subseteq \Omega$. Then, from transcendence, we have $\Omega = \delta(\Omega) \notin \Omega$. From closure, we have $\Omega = \delta(\Omega) \in \Omega$.

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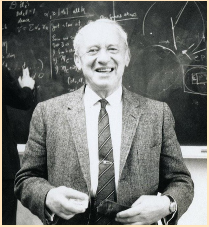
Liar Paradox
Consider $\phi(x)$ as the predicate "$T(x)$", the truth predicate. $\psi(y)$ is the predicate "y is definable". $\delta(y)$ is the function defined by $\delta(y) = \sigma = \ulcorner \sigma \notin y \urcorner$. Then,

- $\Omega = \{x|T(x)\}$ exists and $\psi(\Omega)$ holds.
- Consider the case $\Omega \subseteq \Omega$. Then, from transcendence, we have $\delta(\Omega) \notin \Omega$. From closure, we have $\delta(\Omega) \in \Omega$.

Recap of Truth



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In the context of theories of arithmetic, we attempted to define truth using a formula $\zeta(x)$. This is generalized to arbitrary languages by a predicate $T(x)$ whose inputs are sentence names of the language.

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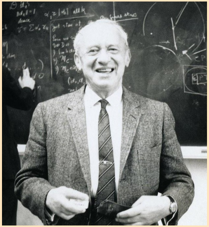

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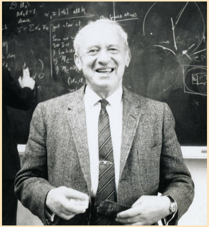

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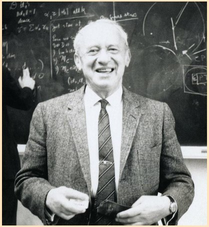

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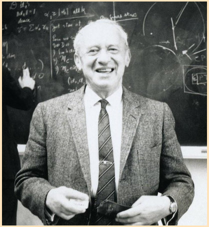

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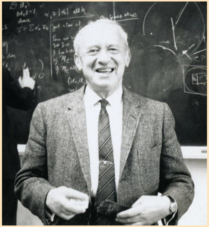

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The axiom schema has $\phi \equiv T(\ulcorner \phi \urcorner)$. Then, we have $T(\ulcorner \phi \urcorner) \equiv \neg T(\ulcorner \phi \urcorner)$. Under classical logic, this entails $T(\ulcorner \phi \urcorner) \wedge \neg T(\ulcorner \phi \urcorner)$.

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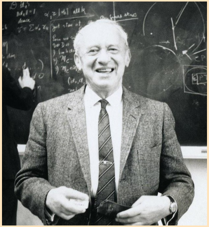

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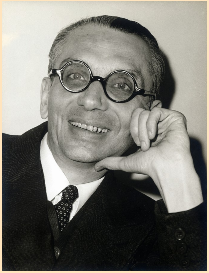
A theory of truth predicates must at least do one of three things: (1) Deny a full truth predicate (2) Admit exceptions to the schema or (3) Change the underlying logic.

Recap of Incompletenesses



Kurt Gödel

Theorem (First Incompleteness)

Any ω-consistent recursively axiomatized extension of $\boldsymbol{Q}$ *is incomplete.*

Theorem (Second Incompleteness)

Let $\boldsymbol{T}$ *be a consistent recursively axiomatizable extension of* $\boldsymbol{Q}$*, set of axioms being* Γ*, then* $\boldsymbol{T}$ *is not derivable from* Γ.

Recap of Epistemic Syntactic Version

In the context of formal epistemology and epistemic logic, we can introduce an predicate K, called the knowability predicate. We adopt a slightly weaker version of the T schema for K, namely,

- $K(\ulcorner\phi\urcorner) \supset \phi$ for all ϕ.
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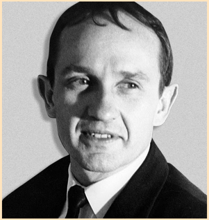


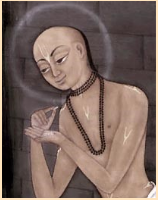
Richard Montague

Theorem

Any formal theory extending $\mathbf{Q}$ *and containing the* K *axiom schemas is inconsistent*

Navya-Nyāya Logic

Udayana



Raghunātha Śiromaṇi

Navya-Nyāya logic

- 14th century to the 20th century
- Synthesis of Nyāya syllogism, debates with Dharmakīrti, Dignāga, Prācīna Nyāya and Vaiśeṣika
- Gaṅgeśa's Tattvacintāmaṇi.
- Three main hotspots of Navya-Nyāya emerged in Vārāṇasi, Mithilā and Navadvīpa.
- Novel, rational and cosmopolitan outlook
- Akin to early modernity (post-Renaissance) in Europe.
- Usually presented in a pseudo-artificial form of Sanskrit - a *formal* language yet not *symbolic*.

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Cognition

Navya-Nyāya logic deals with jñāna. For logic, jñāna must be thought of a particular event of a apprehension/judgement of something.

ghaṭo nīlaḥ

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As the Husserlian wisdom goes, awareness is always awareness *of* something. However, non-existent entities of the impossible kind, do not have any *reference* in Nyāya analyses of jñāna. Thus, a free logic is not the right choice for formalizing Nyāya logics.

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Navya-Nyāya logic deals with jñāna. For logic, jñāna must be thought of a particular event of a apprehension/judgement of something.

Nor does one have any troubles with formalizing jñāna due to their being private entities of epistemic agents. As these jñāna are verbalizable, they become intersubjective. An analysis of the verbalized cognition thus supplants worries about the impossibility of forming a *logic* from Nyāya epistemology.

ghaṭo nīlaḥ

Inference

1. Pratijñā Fm: The mountain has fire on it.
2. Hetu Sm: Because, the mountain has smoke on it.
3. Dṛṣṭānta $\forall x(Sx \rightarrow Fx)$: Whatever has smoke on it, has fire on it.
4. Upanaya $Sm \wedge (Sm \rightarrow Fm)$: The mountain is so, i.e., it has smoke on it as a sign of fire.
5. Nigamana Fm: Thus, the mountain has fire on it.

Content of Cognition

The jñāna are always complex. A cognition of the form ‘The pot is blue’ has sub-elements of being something that is present, having potness and having blueness. In general, one supposes that a jñāna has the structure of being a knowledge of a relation (*saṃsarga*) between a qualificandum (*viśeṣya*) and a qualifier (*viśeṣaṇa*).

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The qualificandum is the thing to which we ascribe a qualifier. It may be the case that for a single cognition, we can formulate different yet equivalent ways to express it where the qualifier and qualificandum interchange positions. The qualifier-qualificandum is the generic presentation of a prototypical cognition. One may think of the qualifier as being superimposed (*ādheya*) onto the substratum (*ādhāra*) or locus (*adhikaraṇa*).

Relations, Adjunct, Subjuncts and Limitors

Four relations are of special importance: (i) inherence (samavāya) (ii) contact or conjunction (saṃyoga) (iii) svarūpa (self-establishing) and (iv) identity (tādātmya). A relation has two aspects, an adjunct and a subjunct.

Relations, Adjunct, Subjuncts and Limitors

A locution of the monadic form "… is wise" denotes some abstract property, wisdom in this case. The loci of this locution are exactly the values of x in x is wise.

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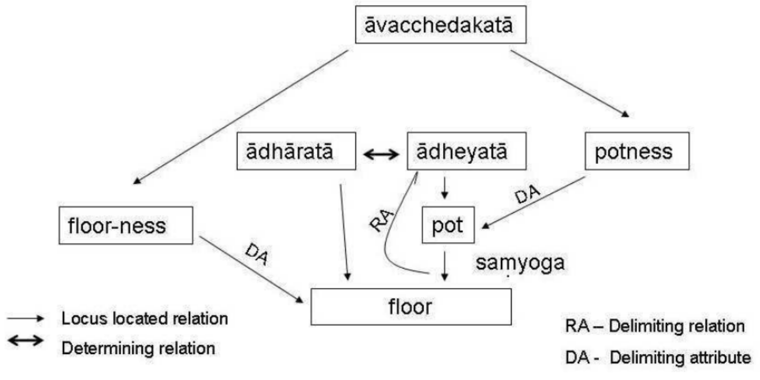
Suppose we had the rather more complex structure of 'x is the mother of y.' We can still use relational abstracts in this case. In particular, these abstracts are conditioned (*nirūpita*) by something else. So, the paraphrasing corresponding to the example here is 'motherhood occurs in x and is condition by y'.

Relations, Adjunct, Subjuncts and Limitors

The concept of limitor or avacchedaka is used in connection with loci. 'In general, a relational abstract residing in an entity may be delimited by the specific relation in which that entity, as a locus of said abstract occurs.'

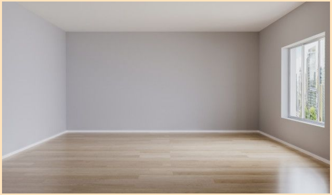
Example



Ghaṭavat bhūtalam (The floor has a pot on it)

Absence

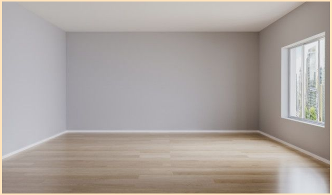
Absence is ontologically serious for the Naiyāyikas. Navya-Naiyāyikas admit four kinds of absences: (i) mutual absence (*anyonyābhāva*) (ii) absence of not-yet (*prāgabhāva*) (iii) absence of no-more (*dhvaṃsābhāva*) and (iv) absolute absence (*atyantābhāva*).



bhūtale ghaṭābhāva

Absence

Negation is always negation *of* something. That somthing, counterpositive is called the *pratyogī*. The counterpositive determines the absence and vice versa.



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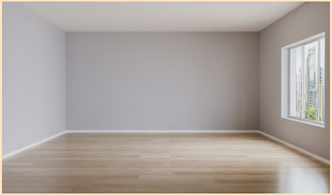
Absence

In particular, Ganeri suggests that if α is a sentence, then the negation of that sentence says something about the anti-object of the sentence α (the absence-of-x say). Following Raghunātha-esque semantics, he suggests the following holds

1. if $\neg T\alpha$ then $T\neg\alpha$
2. $T\neg\neg\alpha$ iff $\neg T\neg\alpha$

In particular, he rejects $T\neg\alpha \rightarrow \neg T\alpha$.



bhūtale ghaṭābhāva

Syntax for Ganeri's Symbolization

Jonardon Ganeri

Jonardon Ganeri proposes a symbolization for a fragment of the Navya-Nyāya technical language.

- First-order setting
- Set-theoretic semantics
- Graph-theoretic semantics for the propositional part
- Extensional logic
- Any addition must be minimal intensional extension.

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9. There are the logical connectives $\wedge, \vee, \rightarrow, \neg$.

Syntax for Ganeri's Symbolization *contd.*

Well-formed formulae or 'sentences' of NN are defined recursively thus. Let a, b, α, β be any terms (primitive and abstract). Let **R**, **S** be relational abstract expressions.

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7. Nothing else.

Semantics for Ganeri's symbolization

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Clarification of Scope

A particular mountain m is the locus of a particular smoke s	$m.\mathbf{L}s$
A particular mountain m is the locus of some smoke	$m.\mathbf{L}\sigma$
A particular mountain m is the locus of all smoke	$\sigma.\mathbf{L}^{-1}m$
Some mountain is the locus of a particular smoke s	$s.\mathbf{L}^{-1}\mu$
Every mountain is the locus of a particular smoke s	$\mu.\mathbf{L}s$
Some mountain is the locus of some smoke	$\mu{:}\mathbf{L}\sigma$
Every mountain is the locus of some smoke or other	$\mu.\mathbf{L}\sigma$
Every smoke is on some mountain or other' corresponds to the sentence	$\sigma.\mathbf{L}^{-1}\mu$

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Restricted quantifiers are essentially just adding a conjunction, i.e., $(\exists x : A)Fx$ is the same as $\exists x(Ax \wedge Fx)$. This identifies NN with a fragment of first order logic with two quantifiers.

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Restricted quantifiers are essentially just adding a conjunction, i.e., $(\exists x : A)Fx$ is the same as $\exists x(Ax \wedge Fx)$. This identifies NN with a fragment of first order logic with two quantifiers.
Consider the example 'Every mountain is the locus of a particular smoke s' - $\mu.\mathbf{L}s$. This is equivalent to $(\exists s : \sigma)(\forall m : \mu)(m\mathbf{L}s)$

Syntax and Semantics for George Bealer's property logic

The language T1 contains the following symbols:

1. A set of variables - $x, y, z, \ldots$
2. A set of constants - $a, b, c \ldots$
3. A set of predicate letters - R^i_j, with i denoting that R^i_j is i-ary
4. A unique 2-ary predicate - $=$
5. Delimiters - (,), [,]
6. Logical connectives - $\wedge$, $\vee$, $\rightarrow$, $\neg$
7. Quantifiers - $\exists$, $\forall$

Syntax and Semantics for George Bealer's property logic

For the first order part, the interpretation is done exactly as done in usual first order logic.
The semantics for the intensional part as such:
A term $[A]_{x_1,\ldots,x_n}$; $n \geq 0$ denotes:

1. A proposition if $n = 0$
2. A property if $n = 1$
3. A n-ary relation if $n \geq 2$

Syntax and Semantics for George Bealer's property logic

Let A be a well-formed formula The modal operators $\Box$, $\Diamond$ are defined as:

$$\Box A := [A] = [[A] = [A]]$$

$$\Diamond A := \neg\Box\neg A$$

$$\Diamond A := \neg([\neg A] = [[\neg A] = [\neg A]])$$

Axiomatic system for Bealer's property logic

The axiomatization of T1 formed by the following axioms is sound and complete with respect to the semantics:

- A1: Propositional tautologies
- A2: $\forall x A(x) \rightarrow A(t)$ where t is free for x in A.
- A3: $\forall x(A \rightarrow B) \rightarrow (A \rightarrow \forall x B)$ where x is not free in A.
- A4: $x = x$ for all variables x.
- A5: $(x = y) \rightarrow (A(x, x) \leftrightarrow A(x, y))$, where the in the formula $A(x, x)$ some but not necessarily all free occurrences of x are replaced by y.
- A6: $\neg([A]_{x_1,\ldots,x_n} = [B]_{y_1,\ldots,y_m})$ where $n \neq m$.
- A7: $[A(x_1, \ldots, x_n)]_{x_1,\ldots,x_n} = [A(y_1, \ldots, y_n)]_{y_1,\ldots,y_n}$ where x_i is replaced by y_i
- A8: $([A]_{x_1,\ldots,x_n} = [B]_{x_1,\ldots,x_n}) \leftrightarrow (\Box \forall x_1 \ldots \forall x_n (A \leftrightarrow B))$
- A9: $\Box A \rightarrow A$
- A10: $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$
- A11: $\Diamond A \rightarrow \Box \Diamond A$

Axiomatic system for Bealer's property logic

The rules of inference are the following:

1. Modus ponens: If $\vdash A$ and $\vdash A \rightarrow B$, then $\vdash B$.
2. Universal quantification: If $\vdash A$, then $\vdash \forall x A$ for any variable x.
3. Necessity introduction: If $\vdash A$, then $\vdash \Box A$.

tattvavat tad eva

The comprehension principle "*tattvavat tad eva*" is an important rule of Navya-Nyāya logic. Essentially, it says "Anything which possesses the property of 'being that' is that." In this sentence, *that* is used as a dummy noun.

Eberhard Guhe

The comprehension principle "*tattvavat tad eva*" is an important rule of Navya-Nyāya logic. Essentially, it says "Anything which possesses the property of 'being that' is that." In this sentence, *that* is used as a dummy noun. Suppose we have a property x-ness. Then the rule *tattvavat tad eva* says, "anything that has the property of x-ness is an x." Guhe suggests the formalization:

$$a\Delta[A(x)]_x \leftrightarrow A(a)$$

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tattvavat tad eva

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Suppose now that A, r is $\neg x\Delta x$.

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$$a\Delta[A(x)]_x \leftrightarrow A(a)$$

According to the rule *tattvavat tad eva*, we have

$$r\Delta[\neg x\Delta x]_x \leftrightarrow (\neg r\Delta r)$$

Now suppose that $r\Delta r$. From modus tollens, we get $r\Delta[\neg x\Delta x]_x$. From modus ponens on this, we get $\neg r\Delta r$.

If instead, we suppose that $\neg r\Delta r$, then from modus ponens on the rule, we get $r\Delta[\neg x\Delta x]_x$. Hence, we have $r\Delta r$.

Eberhard Guhe

Type-Theory to the Rescue

Type-Theory! A hierarchical ontology with many levels - urelements, set-like properties and class-like properties.

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We use lower case bold face letters for urelements, uppercase letters for class-like properties and lower case letters for set-like properties.The relevant extension in our case will be called "T1+".

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The additional axioms of T1+ include the sethood axiom which states that every element of a set is a set or an urelement i.e., $\forall X \forall \mathbf{x}(\mathbf{x} \in X \rightarrow (E\mathbf{x} \vee S\mathbf{x}))$, the axioms of Quine-Morse set theory (except the extensionality axiom) and the axioms of ZF (again except the extensionality axiom).

Theorems for the Naiyāyikas

Guhe suggests a property-theoretic version of the axiom of foundation for the case of Navya-Nyāya logic:

$$\forall X(\exists \mathbf{y}(\mathbf{y}\Delta X) \to \exists \mathbf{y}(\mathbf{y}\Delta X \wedge \forall \mathbf{z}(\mathbf{z}\Delta X \to \neg \mathbf{z}\Delta \mathbf{y})))$$

Theorem

$$\neg\exists(a_n)_{n\in\mathbb{N}}\forall i \in \mathbb{N}(a_{i+1}\Delta a_i)$$

Proof.

Suppose there is a sequence a_n, $n \in \mathbb{N}$, such that $a_{i+1}\Delta a_i$ for all $i \in \mathbb{N}$. Let X be a property whose loci are a_n, i.e., $X = [\exists n \in \mathbb{N}(x = a_n)]_x$. Then, we have $\forall a_i(a_i\Delta X \to a_{i+1}\Delta a_i \wedge a_{i+1}\Delta X)$. Turning the a_{i+1} into an existential quantification, we get $\forall a_i(a_i\Delta X \to \exists \mathbf{z}(\mathbf{z}\Delta a_i \wedge \mathbf{z}\Delta X))$. This is in contradiction with the axiom of foundation proposed above. □

Theorem (De Morgan's Law)

$$[\neg\exists \boldsymbol{y}(F\boldsymbol{y} \wedge \boldsymbol{x}L\boldsymbol{y}) \wedge \neg\exists \boldsymbol{y}(G\boldsymbol{y} \wedge \boldsymbol{x}L\boldsymbol{y})]_{\boldsymbol{x}} = [\neg\exists \boldsymbol{y}((F\boldsymbol{y} \vee G\boldsymbol{y}) \wedge \boldsymbol{x}L\boldsymbol{y})]_{\boldsymbol{x}}$$

Ingalls gives definition of *anyatara*. If **x** is either fire or water (anyatara), then **x** possesses the mutual absence to which the counterpositiveness is limited by a pair or mutual absences, namely the mutual absence of fire and the mutual absence of water.

Proof.

(A1)	$(F\mathbf{y} \vee G\mathbf{y}) \wedge \mathbf{x}L\mathbf{y} \leftrightarrow F\mathbf{y} \wedge \mathbf{x}L\mathbf{y} \vee G\mathbf{y} \wedge \mathbf{x}L\mathbf{y}$
(R2)	$\forall\mathbf{y}((F\mathbf{y} \vee G\mathbf{y}) \wedge \mathbf{x}L\mathbf{y} \leftrightarrow F\mathbf{y} \wedge \mathbf{x}L\mathbf{y} \vee G\mathbf{y} \wedge \mathbf{x}L\mathbf{y}$
(FOL)	$\exists\mathbf{y}((F\mathbf{y} \vee G\mathbf{y}) \wedge \mathbf{x}L\mathbf{y}) \leftrightarrow \exists\mathbf{y}(F\mathbf{y} \wedge \mathbf{x}L\mathbf{y} \vee G\mathbf{y} \wedge \mathbf{x}L\mathbf{y})$
(FOL)	$\exists\mathbf{y}((F\mathbf{y} \vee G\mathbf{y}) \wedge \mathbf{x}L\mathbf{y}) \leftrightarrow \exists\mathbf{y}(F\mathbf{y} \wedge \mathbf{x}L\mathbf{y}) \vee \exists\mathbf{y}(G\mathbf{y} \wedge \mathbf{x}L\mathbf{y})$
(A1)	$\neg\exists\mathbf{y}((F\mathbf{y} \vee G\mathbf{y}) \wedge \mathbf{x}L\mathbf{y}) \leftrightarrow \neg(\exists\mathbf{y}(F\mathbf{y} \wedge \mathbf{x}L\mathbf{y}) \vee \exists\mathbf{y}(G\mathbf{y} \wedge \mathbf{x}L\mathbf{y}))$
(De Morgan)	$\neg\exists\mathbf{y}((F\mathbf{y} \vee G\mathbf{y}) \wedge \mathbf{x}L\mathbf{y}) \leftrightarrow \neg\exists\mathbf{y}(F\mathbf{y} \wedge \mathbf{x}L\mathbf{y}) \wedge \neg\exists\mathbf{y}(G\mathbf{y} \wedge \mathbf{x}L\mathbf{y})$
(R2, R3)	$\Box\forall\mathbf{x}(\neg\exists\mathbf{y}((F\mathbf{y} \vee G\mathbf{y}) \wedge \mathbf{x}L\mathbf{y}) \leftrightarrow \neg\exists\mathbf{y}(F\mathbf{y} \wedge \mathbf{x}L\mathbf{y}) \wedge \neg\exists\mathbf{y}(G\mathbf{y} \wedge \mathbf{x}L\mathbf{y}))$
(A8, R1)	$[\neg\exists\mathbf{y}(F\mathbf{y} \wedge \mathbf{x}L\mathbf{y}) \wedge \neg\exists\mathbf{y}(G\mathbf{y} \wedge \mathbf{x}L\mathbf{y})]_{\mathbf{x}} = [\neg\exists\mathbf{y}((F\mathbf{y} \vee G\mathbf{y}) \wedge \mathbf{x}L\mathbf{y})]_{\mathbf{x}}$

$\square$

Theorems for the Naiyāyikas

1. $o := [\neg\exists \mathbf{y}(\mathbf{y}\Delta\mathbf{x})]_{\mathbf{x}}$
2. $\mathbf{x}^{!} := [\exists \mathbf{u}(\mathbf{u}\Delta\mathbf{x} \wedge \exists \mathbf{v}(\mathbf{v}\Delta\mathbf{u} \wedge \mathbf{y} = [\mathbf{w}\Delta\mathbf{u} \vee \mathbf{w} = \mathbf{v}]^{\mathbf{uv}}_{\mathbf{w}}))]^{\mathbf{x}}_{\mathbf{y}}$
3. $NN\mathbf{x}$ ($\mathbf{x}$ is a natural number) if and only if $\forall \mathbf{z}(o\Delta\mathbf{z} \wedge \forall \mathbf{y}(\mathbf{y}\Delta\mathbf{z} \rightarrow \mathbf{y}^{!}\Delta\mathbf{z}) \rightarrow \mathbf{x}\Delta\mathbf{z})$

The superscript after the square bracket represents that the variables are free. The standard axioms then are formalizable in this system in the obvious way. Peano's fifth axiom of induction can also be included in the system as the following

$$\forall \mathbf{z}(o\Delta\mathbf{z} \wedge \forall \mathbf{x}(\mathbf{x}\Delta\mathbf{z} \rightarrow \mathbf{x}^{!}\Delta\mathbf{z}) \rightarrow \forall \mathbf{x}(NN\mathbf{x} \rightarrow \mathbf{x}\Delta\mathbf{z}))$$

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